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Procedia Engineering 96 (2014) 320 – 329

Procedia Engineering

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Modelling of Mechanical and Mechatronic Systems MMaMS 2014

The Synthesis of a Hook Drive Cam Mechanism

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Abstract

Cam mechanisms are very important mechanical objects which are used in the design of various machinery. Their application can be found, for example, in machine tools, textile, printing, sewing machines, etc. This mechanical object was used in the design of a sewing machine hook drive. The hook drive is composed by a four-bar linkage and a cam mechanism with radial conjugate cams and an oscillating roller follower. The main aim of the kinematic synthesis was to determine the optimal characteristic parameters of the cam mechanism hook drive. As the criterion for the determining of the most appropriate dimensions, it was chosen the pressure angle course in the proposed mechanical system. Another goal of the computational analysis was the determining of the cam mechanism dynamic behavior and properties, which are expressed in the Hertzian pressure course and estimated rating life of curve roller bearings.

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Keywords: Cam mechanism; Displacement law; Kinematic synthesis; Pressure angle; Hertz pressure

1. Introduction

Cam mechanisms may implement required working motion very precisely with a small number of bodies located in a relatively small space, thus it is used in the design of varied machinery. Cam mechanisms have an influence over dynamic properties and behavior of the given mechanical system. The influence depends on the choice of a displacement law which expresses a functional dependency of the driven member motion on the driving member motion of a mechanical system.

Increasing requirements on the performance of the mentioned machines demand appropriate approach for the design of cam mechanisms. With the development of numerical mathematics and informatics, numerical methods have begun

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Peer-review under responsibility of organizing committee of the Modelling of Mechanical and Mechatronic Systems MMaMS 2014
doi:10.1016/j.proeng.2014.12.129

to be applied in the design of cam mechanisms. At present, it is possible to implement user-oriented subroutines into commercially available expert systems which describe mathematic models of displacements and kinematic quantities of cam mechanisms and transforming linkages. The set of these subroutines may be used for kinematic analysis and synthesis of cam mechanisms. They may be also applied to the dimensional optimization of the mechanism regarding e.g. the value of the minimal curvature radius of the cam profile or the maximum pressure angle. They may be used to solve the dynamics of combined cam mechanisms with rigid or flexible bodies.

2. Combined cam mechanisms

This section gives only basic information on how to solve tasks of combined cam mechanisms. Detailed knowledge of this issue may be found in [1].

2.1. General cam mechanisms

The general cam mechanism is typically referred to as a three-link mechanism with single degree of freedom, which includes at least one cam connected to the driven link with at least one general kinematic pair. The driving member of a cam mechanism is the cam. In terms of shape, it is possible to define the basic cam types: radial, axial (cylindrical), globoid, segmented and conical. The driven member of a cam mechanism is the follower, which performs the desired motion. The translating follower motion is defined as translating or general. A rotating follower is usually called the lever.

The cam profile coordinates are determined by the synthesis which is implemented on the basis of the displacement law knowledge of the given cam mechanism and its dimensional parameters. The cam position relative to the cam mechanism frame is defined with an angular variable ψ and the follower position is indicated with the generalized variable v. The solving algorithms of general cam mechanisms mathematical models include relevant subroutines and that are identified by block *C* for other needs.

Schematic representation of a cam mechanism with radial conjugate cams and an oscillating roller follower is shown in Fig. 1.

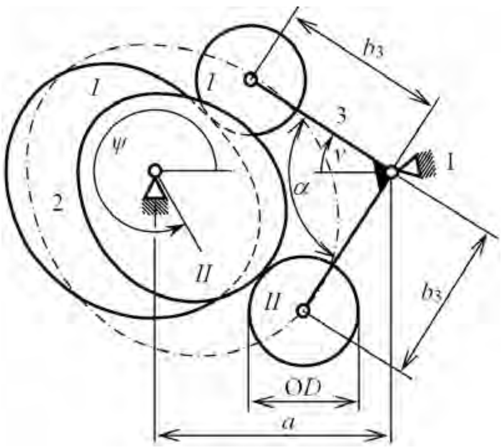

Fig. 1. General cam mechanism with radial cams and a lever.

2.2. Combined cam mechanisms

As a combined cam mechanism, it is generally called a mechanical system usually with single degree of freedom which includes at least one general cam mechanism. The system may further includes gearing and linkages with not only a constant, but also a generally variable gear ratio.

The relevant linkage input is an ordered triple of variables $\mathbf{x}_k = (x_k, \dot{x}_k, \ddot{x}_k)$ expressing motion of the input link of a mechanical system. The output is then a triple of variables $\mathbf{y}_k = (y_k, \dot{y}_k, \ddot{y}_k)$, which represents the motion of the output link of the same system. Index k denotes the numerical indication of the relevant linkage. Index k will be neglected in the next text part. The general equation of the linkage may be defined as an implicit function:

$$F(x,y)=0, \qquad x=x(t), \qquad y=y(t). \tag{1}$$

By differentiating the equation (1) in time, a relation may be obtained between velocity and acceleration [1]:

$$\frac{\partial F}{\partial x}\dot{x}+\frac{\partial F}{\partial y}\dot{y}=0, \qquad \frac{\partial F}{\partial x}\ddot{x}+\frac{\partial^2 F}{\partial x^2}\dot{x}^2+2\frac{\partial^2 F}{\partial x \partial y}\dot{x}\dot{y}+\frac{\partial^2 F}{\partial y^2}\dot{y}^2+\frac{\partial F}{\partial y}\ddot{y}=0. \tag{2}$$

Differentiation with respect to time is denoted by dots in equations (2). Linkage ratio is called a magnitude, which is dependent on the position of the linkage, and it is given by equation (3):

$$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{\partial F}{\partial x}\left(\frac{\partial F}{\partial y}\right)^{-1}. \tag{3}$$

The derivative of equation (3) with respect to positions is given by equation (4):

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}=-\left[\frac{\partial^2 F}{\partial x^2}+2\frac{\partial^2 F}{\partial x \partial y}\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{\partial^2 F}{\partial y^2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]\left(\frac{\partial F}{\partial y}\right)^{-1}. \tag{4}$$

Introducing equation (3) and (4) into equations (2) gives

$$\dot{y}=\frac{\mathrm{d}y}{\mathrm{d}x}\dot{x}, \qquad \ddot{y}=\frac{\mathrm{d}y}{\mathrm{d}x}\ddot{x}+\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\dot{x}^2, \tag{5}$$

which express relations for calculating velocity and acceleration of the output member. Equations (1) to (5) represent the solving algorithms of the linkage mathematical models, which are defined by relevant subroutines.

The individual linkage can be placed into so-called *chains* when the outputs of the given mechanism are also the inputs of the next mechanism. The algorithms of linkage solving are marked with transformation blocks T_k, as shown in Fig. 2.

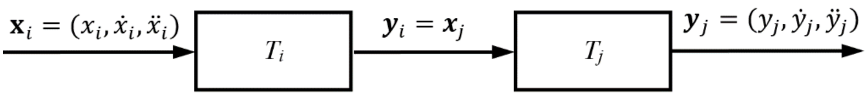

Fig. 2. Linkage chain.

To the driving link of a combined cam mechanism, it is assigned variable τ. The driven link position is indicated with the variable w. For a driven link, it is usually regarded as a working link or the body whose values are the solution result.

2.3. Displacement laws

A displacement law expresses a functional dependency of the driven body motion on the driving member motion of a combined cam mechanism. Displacement laws $w(\tau)$ of mechanical systems with rotating cams are periodical functions with a period of 2π. The period 2π may be divided into motion and dwell intervals, see Fig. 3. Displacements on each motion interval may be different in a maximum total rise W and an expression of the normalized form where the displacement $\eta = \eta(\xi)$ and the range are unity [1].

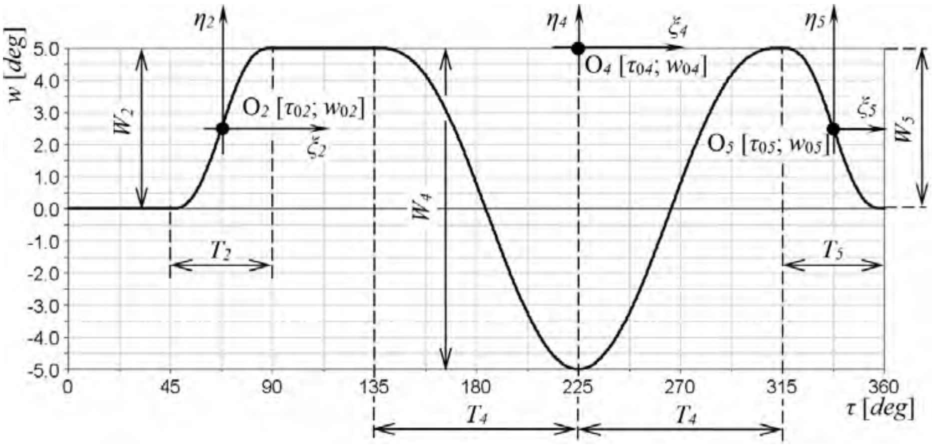

Fig. 3. Displacement law.

The variables ξ, η of each motion interval are in linear correlation with the original variables τ, w and can be expressed as [1]:

$$\xi = \frac{(\tau - \tau_0)}{T}, \qquad \eta(\xi) = \frac{(w(\tau) - w_0)}{W}. \tag{6}$$

The initial point of displacement on the motion interval is defined by the coordinates τ_0, w_0. The interval length of the independent variable τ is given by the magnitude T. The relationship between the original and normalized derivatives is:

$$\frac{\mathrm{d}w}{\mathrm{d}\tau} = \frac{W}{T}\frac{\mathrm{d}\eta}{\mathrm{d}\xi} = \frac{W}{T}\eta'(\xi), \qquad \frac{\mathrm{d}^2 w}{\mathrm{d}\tau^2} = \frac{W}{T^2}\frac{\mathrm{d}^2\eta}{\mathrm{d}\xi^2} = \frac{W}{T^2}\eta''(\xi). \tag{7}$$

Derivatives of the unity displacement to ξ will be denoted by primes. When the variable w is a function time t, then the derivatives will have the form:

$$\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{W}{T}\eta'(\xi)\frac{\mathrm{d}\tau}{\mathrm{d}t}, \qquad \frac{\mathrm{d}^2 w}{\mathrm{d}t^2} = \frac{W}{T}\left[\frac{1}{T}\eta''(\xi)\left(\frac{\mathrm{d}\tau}{\mathrm{d}t}\right)^2 + \eta'(\xi)\frac{\mathrm{d}^2\tau}{\mathrm{d}t^2}\right]. \tag{8}$$

To the solution of problems related to the kinematic analysis and synthesis of cam systems, it is possible to use a broad set of displacement laws in a normalized form. These include, for example, polynomial, trigonometric and exponential displacements, cycloidal, parabolic and goniometric displacement. The user library of the displacement laws on the basis of equations (6) to (8) was developed.

2.4. Solving tasks of combined cam mechanisms

The procedure of calculating a combined cam mechanism is shown through a block diagram, which is formed from transform blocks T_k and transform block C, see Fig. 4. From the view of the structure of a block diagram, it is advisable to divide the transform blocks into three groups, which are identified by the term *chain*:
- The input *chain IC* connects the input of the system with the input of block *C*,
- The output *chain OC* connects the output of block *C* with the output of the system,
- The parallel *chain PC* consists of blocks that do not belong either to *IC* input chain or *OC* output chain.

Indexes *i*, *j*, *k*, *l* denote i^{th}, j^{th}, k^{th}, l^{th} the linkage of the relevant chain.

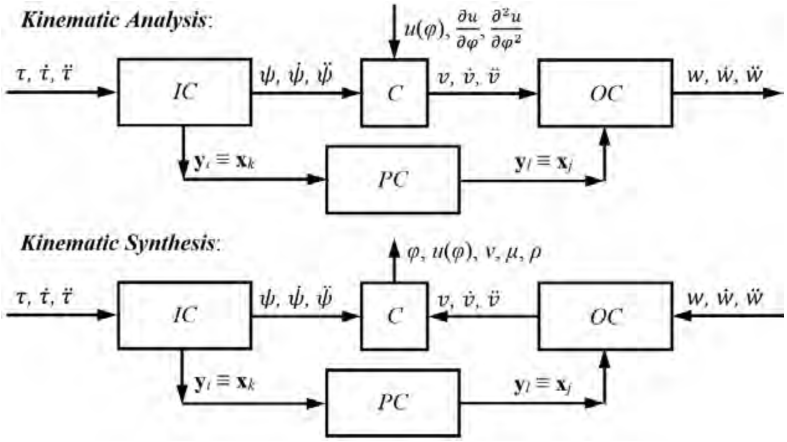

Fig. 4. Block diagram of a combined cam mechanism computation.

The calculation of the positional and kinematic quantities of any link of a combined cam mechanism is designed as kinematic analysis, see the block diagram in Fig. 4. The input data are typical data on a mechanism, the procedure of computation of its chains, displacement function $\tau = \tau(t)$ of the driving link and the cam theoretical profile coordinates $u(\varphi)$. The output of the solution is the displacement of the selected link of the mechanical system w, $\dot{w}$, $\ddot{w}$.

As a kinematic synthesis of a combined cam mechanism, it is designed the computation of polar coordinates φ, u of a radial cam or cylindrical coordinates φ, x, y of an axial cam. Furthermore, via synthesis, normal angle v, pressure angle μ and radius of curvature of cam profile ρ are set. The input data of the task include data on a mechanical system, the calculation procedure of its chains, displacement function of driving link $\tau = \tau(t)$ and driven link $w = w(t)$ of the system.

3. The hook drive cam mechanism

The subject of the development work has been the optimal characteristic parameters determining of the cam mechanism hook drive. As the criterion for the determining of the most appropriate dimensions, it was chosen the pressure angle course μ. The hook drive itself consists of a four-bar mechanism and a cam mechanism with radial conjugate cams and an oscillating roller follower, as shown in Fig. 5.

The basic kinematic scheme of a sewing machine hook drive and the geometrically-mass characteristics of the individual members of the four-bar linkage were specified. The four-bar linkage is defined as implicit function [1]:

$$L(\sigma,\tau) = l^2 - b^2 - p^2 - q^2 + 2pq\cos(\sigma - \tau) + 2b(p\cos\sigma - q\cos\tau) = 0. \tag{9}$$

The four-bar mechanism links are crank *2*, connecting rod *3* and radial conjugate cams *4*.

The solving algorithms of problems related to the kinematic synthesis of the given cam mechanism are expressed as [1]:

$$
\begin{aligned}
u &= \sqrt{a^2 + b_{A,B}^2 - 2ab_{A,B}\cos w},\\
\varphi &= \psi + \arccos\frac{a - b_{A,B}\cos w}{u},\\
\mu &= -\arctan\frac{\dot{\psi}\left(a\cos w - b_{A,B}\right) + \dot{w}b_{A,B}}{\dot{\psi}\alpha\sin w}.
\end{aligned}
\tag{10}
$$

Equations (10) represent the computation of polar coordinates φ, u of the radial cam and pressure angle μ.

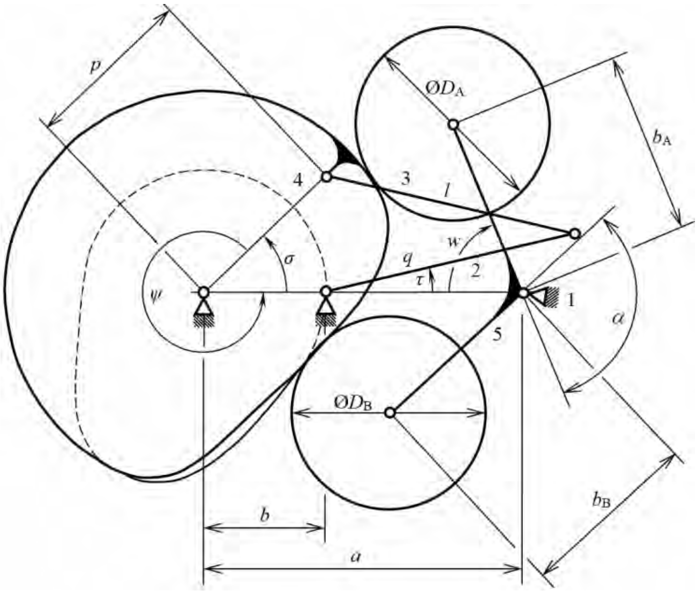

Fig. 5. Kinematic scheme of the hook drive.

The aim of the optimization calculations was to determine the characteristic dimensions of the cam mechanism, i.e.:

- distance a of rotary axes of double cam and lever,
- arm lengths b_A and b_B of lever, while $b_A = b_B = b$,
- angle α between the arms of lever

and a shape of the radial conjugate cams.

The driving member of the mechanical system is crank *2*, its position relative to the frame is determined with the angular variable τ. The driven body is the cam mechanism lever *5*, its position is indicated with the angular variable w. The dynamic properties and behavior of the hook drive were assessed on the basis of a displacement law $w(\tau)$ whose motion intervals are described by a polynomial of the fifth degree [1]:

$$
\eta(\xi) = \frac{1}{8}\xi\left[15 - 10(2\xi)^2 + 3(2\xi)^4\right].
\tag{11}
$$

The initial and the final point of each motion intersection end the maximum rise W are defined in Table 1. The displacement law course is represented in Fig. 6.

Table 1. Boundary points of displacement law.

Boundary points	*1st*	*2nd*	*3rd*	*4th*	*5th*
τ (*deg*)	0	223	263	287	360
$W_{\max}$ (*deg*)	0	0	28	28	0

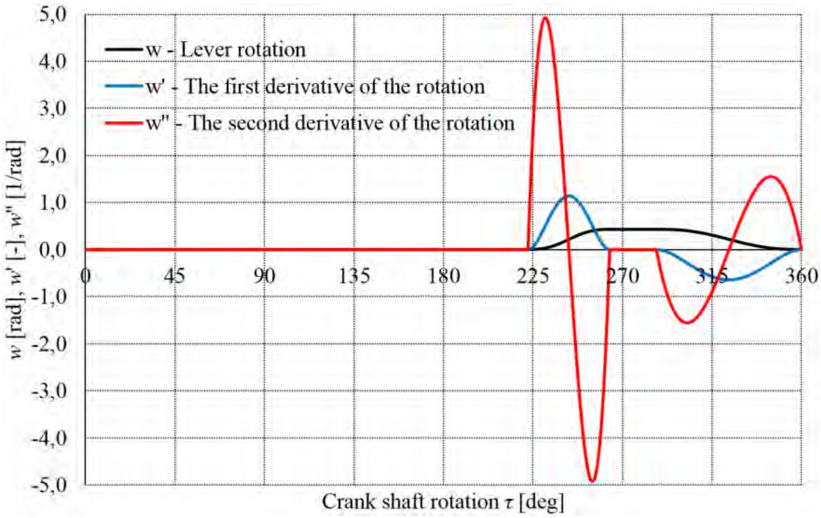

Fig. 6. Displacement law.

The resulting mathematical model of the hook drive combined cam mechanism was created on the basis of the optimization processes of the cam mechanism synthesis in such way so that to achieve the smallest possible value of pressure angle μ. The own synthesis was carried out in the environment of expert systems NX I-DEAS, MSC.ADAMS and MSC.EASY5.

3.1. A sample of the results

The pressure angle μ expresses the cam force effect on the driven member of a cam mechanism. It is the angle between the normal to the pitch profile of the cam and the direction of the follower motion. This angle is important in cam design because it represents the cam profile steepness. The smaller the value, the more favorable transmission of force between the cam and the cam mechanism driven body. It would be advisable the pressure angle to be between zero and about ±35 *deg* for translating followers to avoid excessive side load on the sliding follower. If the follower is oscillating on a pivoted arm, a pressure angle up to about ±30 *deg* is acceptable.

Several variants of the cam mechanism were analyzed, wherein pressure angle extreme values of some of these are presented in Table 2. The characteristic dimensions of the cam mechanism are not given for reasons of confidentiality.

Table 2. Cam mechanism pressure angle.

Cam mechanism pressure angle	*Variation V1*	*Variation V2*	*Variation V3*
$\lvert\mu_A\rvert$ (*deg*)	42.0	27.8	26.0
$\lvert\mu_B\rvert$ (*deg*)	42.5	28.4	25.4

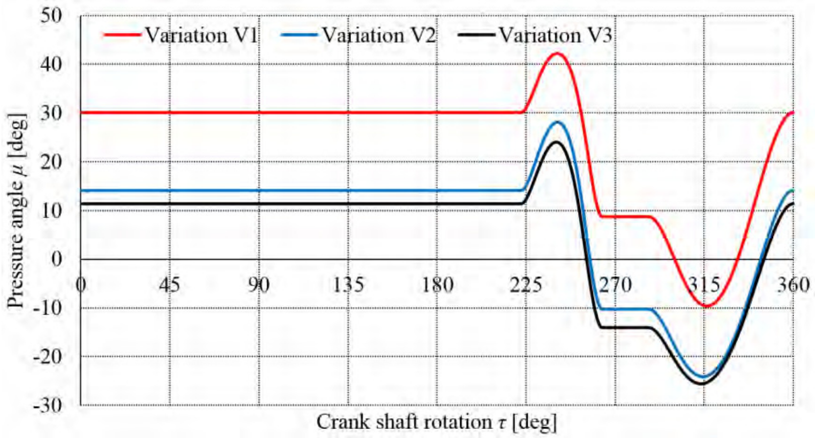

Fig. 7. Pressure angle of the cam mechanism A.

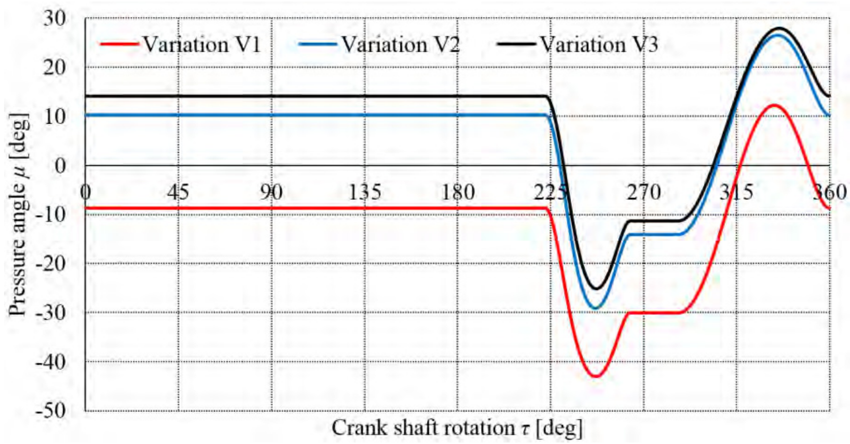

Fig. 8. Pressure angle of the cam mechanism B.

In the operation of cam mechanisms, fatigue damage of the contact surfaces of a cam and a follower may appear. This damage is in the form of pitting that develops from cracks on the surface of the active area. This type of damage is due to contact stress described by Hertz theory. The contact of the roller with the cam occurs in the contact surface of length l whereas in its length, a transfer of normal reaction N occurs in the general kinematic pair. Normal reaction N will cause a deformation of both bodies and induce contact pressure p. Maximum pressure value p is called Hertz pressure, which can be determined from the relationship derived from the contact theory of cylindrical bodies with parallel axes [1, 2]:

$$p_H = \sqrt{\frac{N\left(|\rho| + sign(\rho) R_K\right)}{\pi(\delta_1 + \delta_2) l R_K |\rho|}}, \qquad \delta_{1,2} = \frac{1 - \nu_{1,2}^2}{E_{1,2}}. \tag{12}$$

R_k denotes the roller diameter and ρ the radius of curvature of the active surface of the cam. Quantities $\delta_{1,2}$ characterize the elasticity of the material of the bodies in contact. Constants $\nu_{1,2}$ represent Poisson's ratio and $E_{1,2}$ Young's modulus of elasticity in tension. The value of the permissible Hertz pressure standardly ranges from $p_H = 1200$ up to 1400 MPa.

This value depends on the grade of steel from which a cam is produced and on the heat or chemo-thermal treatment of the surface layer of cams. Fig. 5 shows a possible course of Hertz pressures in the cam mechanism of the hook drive at the operating frequency n_n = 2000 *rev/min*. From their curves it is evident that the value of Hertz pressure within the machine working cycle does not exceed the size limit.

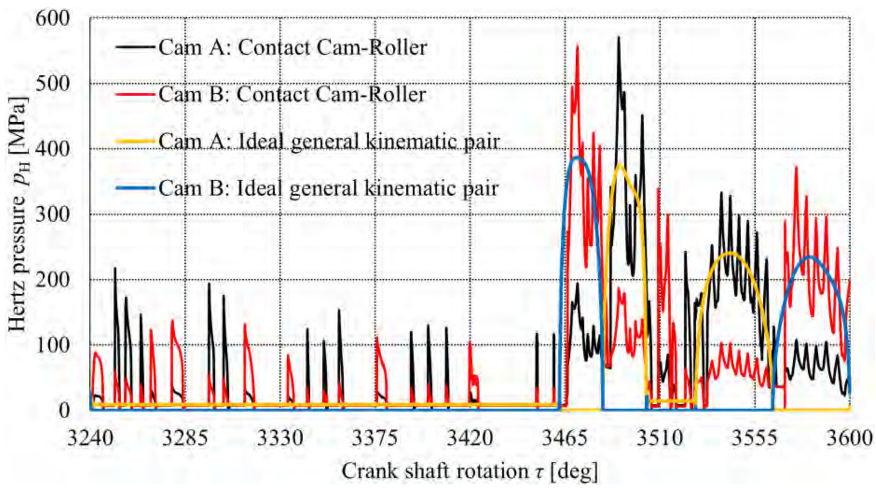

Fig. 9. Hertz pressure.

In terms of the dynamic behavior of the analyzed mechanic system, rating life L_{10} of the bearing of the cam mechanism rollers was assessed that result from the relationship [3]:

$$L_{10} = \left(\frac{C_W}{R_{ekv}}\right)^p \cdot 10^6 \qquad [\text{cycles}]. \tag{13}$$

In expression (13), C_w stands for the basic dynamic load rating of the bearing and R_{ekv} for equivalent load, which is determined on the basis of reaction forces in the respective bearing. The parameter of the bearing p achieves those values:

- $p = 3$: for bearings with point contact,
- $p = 10/3$: for bearings with line contact.

Equivalent load is determined using the relation:

$$R_{ekv} = \sqrt[p]{\frac{\sum_i \omega_i R_i^p}{\sum_i \omega_i}} \qquad [N], \tag{14}$$

where R_i, or ω_i, are discrete values of reaction in the bearing or the angular velocity of the respective bearing during one working cycle. For the machine working speed n_n = 2000 *rev/min*, it results the rating life of the bearings of the cam mechanism rollers according to Table 3.

Table 3. Rating life of the bearings of the rollers.

Cam roller	p	C_w (*N*)	R_{ekv} (*N*)	$L_{10} \cdot 10^6$ (*cycles*)
Roller A	10/3	3140	110	38000
Roller B	10/3	3140	120	59000

4. Conclusions

User libraries of procedures and functions have been created that contain the solving algorithms of combined cam mechanism mathematical models. These subroutines describe mathematical models of displacements and kinematic quantities of cam mechanisms and transforming linkages. They are used for an efficient creation of mathematical models of those mechanical systems.

This approach of designing a cam mechanism was applied in the synthesis of the sewing machine hook drive. The subject of the development work has been the determining of the optimal characteristic dimensions of the cam mechanism hook drive. As the criterion for the determining of the most appropriate dimensions, it was chosen the pressure angle course μ. The pressure angle μ expresses the cam force effect on the cam mechanism driven member. The dynamic properties and behavior of the hook drive were assessed on the basis of a displacement law whose motion intervals are described in a polynomial of the fifth degree.

Acknowledgements

This paper was created within the work on the FR-TI4/271 – Project supported by the Ministry of Industry and Trade.

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